

Flexible Robot Arm Control Using Adaptive Control Structure

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Abstract – The use of various robotic arm structures in industries is increasing. Industrial robots are a key component of most modern automation systems. In this paper, we study the dynamic modeling and control of flexible joint arm robots. In this regard, the dynamic model of the robot in the form of state space will be extracted using the Lagrange-Euler method. A new adaptive backstepping control approach is introduced to track the reference and improve the robustness of the nonlinear system. By using this method, changing the position and parameters of the robot will not affect the robust performance of the proposed controller. To analyze the results of this method uses MATLAB software and according to the desired trajectory for the robot arms, the adaptive robust control input for the system is obtained. Based on results the proposed control structure controls the model property without overshoot and by considering the saturation limit of input control.

Keywords: Flexible robot – Nonlinear model-adaptive controller.

1. Introduction

Accurate information on the kinematics and dynamics of industrial arms plays an important role in their design and control. Many researchers have investigated the adaptive control of the flexible flexibility of the robot. Adaptive robot control issues based on a variety of designs, including neural networks, adaptive methods, and nonlinear control, have emerged in recent decades. In [1], an adaptive sliding mode control scheme (ASMC4) is presented, which is applied to the robot arm using the linear estimation method. ASMC proposes using a new adaptive law to achieve good system tracking performance. In [2], an adaptive controller for the control of the humanoid robot with flexible joints is presented.

The proposed controller estimates the nonlinear conditions in robot dynamics due to the flexibility of the joints. In [3], a robust, proportional-derivative control scheme is presented for flexible-based robots based on the perturbation observer. The proposed disturbance-based observer performance guarantees the stability of the global correlation model's parameters. In [4], we investigate the problem of path tracking and stabilization of mobile robot

control with uncertain parameters due to input torque saturation and external disturbance. In [5], an adaptive hybrid controller for flexible joints robots has been developed with unknown algebraic parameters, and since a full-time dynamic model has been used in its design and stability analysis, it no longer needs any constraints on joint stiffness. In [6], two robust controllers are proposed to control the robot arm with saturated inputs. Based on previous works, this paper's approaches provide a universal solution to the problem through static feedback. In [7], the problem is to design an adaptive-robust control for the flexible arm of the robot arm by considering the reduction of the number of flexible system dynamics. In [7,8], numerical simulations have been performed for flexible joints, to show the advantages of new adaptive schemes compared to the adaptive's lows algorithms used in [2]. In [8], a controller has been designed for robot arms without speed measurement. It considers an adaptive controller for state control in which the gravity torque is unknown and uncertain. The proposed adaptive control ensures both semi-global convergence and zero-position error based on driving constraints. In [9], motion tracking control, for a class of robot arm with flexible joint, is done by the DC motor control. An observer has been designed to estimate velocity signals. In [10], the result of adaptive control is presented for the flexible joint of the robot arm. In these papers, assume weak joint elasticity model and And nonlinear semesters have been omitted and ignored.

In this paper, the proposed controller aims to follow the desired path for a two-joint flexible robot model with two

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DC motors. The control method includes an adaptive backstepping controller that According to Lyapunov's theory, the stability of the model is guaranteed. The rest of the paper is organized as follows. In the second section, the model of the robot and its challenges are presented. In the third section, the proposed adaptive backstepping controller is designed. In the fourth section, the nonlinear model of the robot is simulated in MATLAB software, and based on results, the performance of the method is evaluated. In the fifth section, according to the evaluations performed, the conclusion of the paper is explained.

2. Flexible Robot

The two-joint flexible robot structure is shown in Fig. 1 with two motors attached to each arm. In this robot the arm movement is elastic and its effect is in the form of a spring.

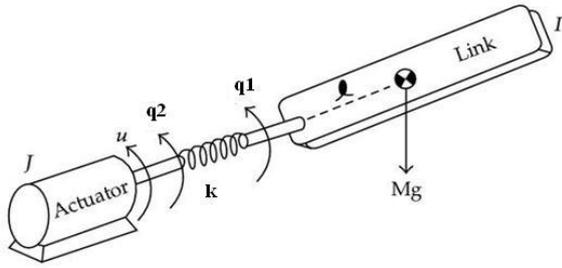


Fig.1. Structure of FJR robot with single link

The dynamic equation of the FJR robot for the n-joints state is in the form of the following relationships, which include the joint rotation relationships and the motor rotation angle of the joints[10]:

$$Mq_1\ddot{q}_1 + Cq_1 \cdot \dot{q}_1 \dot{q}_1 + Gq_1 = Kq_1 - q_2 \quad (1)$$

$$J\ddot{q}_2 + B\dot{q}_2 + Kq_2 - q_1 = \tau \quad (2)$$

In the above relation, q_1 and q_2 are the vectors of the joint angle and the motor rotation angle, respectively. The matrix M, G, C , and B are the general component of inertia, the Coriolis vector, the gravity vector, and the actuator damping component, respectively. K indicates the coefficient of elastic force. Also in the motor equation, the motor inertia matrix (J) and the drive torque matrix (τ) are considered. In single robot mode, the above equations will be in scalar values. To simplify the design of the state space controller the equations of the robot can be obtained by taking the following $[x_1 x_2 x_3 x_4] = [q_1 \dot{q}_1 q_2 \dot{q}_2]$

$$\begin{cases} \dot{x}_1 = x_2, \\ x_1(\dot{x}_1)\dot{x}_2 = -Cx_1\dot{x}_1x_2 - Gx_1 + Kx_1 - x_3 \\ \dot{x}_3 = \\ \dot{x}_4 = -Bx_4 + Kx_1 - x_3 + \tau \end{cases} \quad (3)$$

Note that these equations apply to the articulated robot model, and in the case of two articulated robots in this paper, the system dynamics vector is assumed to be 2×1 . For example, $x_1 = [q_{11} q_{12}]^T$ which means q_{11} and q_{12} are the angles of rotation of the first joint and the second link, respectively. Also for other dynamics $x_2 = [\dot{q}_{11} \dot{q}_{12}]^T$ are the velocity of link $x_3 = [q_{21} q_{22}]^T$ and $x_4 = [\dot{q}_{21} \dot{q}_{22}]^T$. In this paper, the FJR robot parameter are expressed as follows [4]:

$$\begin{aligned} Mq_1 &= \begin{bmatrix} m_1 + m_2 l_1^2 & m_1 l_1 l_2 & s_1 s_2 + c_1 c_2 \\ m_2 l_1 l_2 & s_1 s_2 + c_1 c_2 & m_2 l_2^2 \end{bmatrix}, \\ Cq_1 \cdot \dot{q}_1 &= m_2 l_1 l_2 \begin{bmatrix} 0 & -\dot{q}_2 \\ -\dot{q}_1 & 0 \end{bmatrix}, \\ Gq_1 &= \begin{bmatrix} -m_1 + m_2 & l_1 g s_1 \\ & -m_2 l_2 g s_2 \end{bmatrix}, \\ K &= \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}, \\ B &= \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}, \\ J &= \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}, \end{aligned} \quad (4)$$

where for the sake of simplicity, $c_1 = \cos(q_{11})$, $s_1 = \sin(q_{11})$, $c_2 = \cos(q_{12})$, $s_2 = \sin(q_{12})$.

3. Controller design

In this paper, a backstepping control structure is designed to track and the flexible robot's motion trajectory. Based on (3) the model of the robot system is nonlinear and without eliminating of nonlinear terms of the model, it is necessary to use nonlinear controllers. According to the system equations, to design the backstepping controller, need four steps design for this model. In the first step, the first loop error is defined as follows:

$$e_1 = X_1 - X_{1d} \quad (5)$$

Where x_{1d} is the desire value of x_1 . In the above relation, the virtual control input of step 1 can be considered by the feedback mode by omitting the expression \dot{x}_{1d} :

$$u_1 = -k_1 e_1 + \dot{X}_{1d} \quad (6)$$

the second step of the controller with the relationships from the second loop is given:

$$M\dot{x}_2 = -Cx_2 - G + Kx_3 - x_1 \quad (7)$$

Now divide the sides by K and consider the following parameters:

$$\begin{aligned} MK^{-1} &= M_1 \\ CK^{-1} &= C_1 \\ GK^{-1} &= G_1 \end{aligned} \quad (8)$$

The relation (7) is transformed as follows:

$$\begin{aligned} M_1\dot{X}_2 + C_1X_2 + G_1 &= X_3 - x_1 \\ M_1\dot{X}_2 &= -CX_2 - G_1 + X_3 - X_1 \end{aligned} \quad (9)$$

Now concerning (9), the value of x_3 is inserted:

$$\begin{aligned} M_1\dot{e}_2 &= -C_1e_2 + u_1 - G_1 + e_3 + u_2 - e_1 + x_{1d} - \\ M_1\dot{u}_1 & \\ M_1\dot{e}_2 &= -C_1e_2 - G_1 - \chi_{1d} + e_3 + u_2 - M_1\dot{u}_1 - C_1u_1 - \\ & - e_1, M_1\dot{u}_1 + C_1u_1 + G_1 = y_r\alpha_r \end{aligned} \quad (10)$$

In this section, by defining the matching law for model parameters in the form of a regressor vector, α_r is defined as the vector of undefined parameters and y_r as the known regression matrix that shown in Table 1.

Table 1. Flexible robot measurable parameters

Parameter	Parameter
$y_{11} = \dot{q}_{11}$	$y_{21} = \dot{q}_{12}$
$y_{12} =$	y_{22}
$\dot{q}_{11}[(\sin(q_{11})\sin(q_{12}) +$	$= \dot{q}_{12}[(\sin(q_{11})\sin(q_{12})$
$\cos(q_{11})\cos(q_{12})) -$	$+ \cos(q_{11})\cos(q_{12}))$
$(\sin(q_{11})\cos(q_{12}) +$	$- (\sin(q_{11})\cos(q_{12})$
$\cos(q_{11})\cos(q_{12}))]+$	$+ \cos(q_{11})\cos(q_{12}))]$
$q_{11}[\sin(q_{11})\cos(q_{12}) +$	$+ q_{12}[\sin(q_{11})\cos(q_{12})$
$\cos(q_{11})\cos(q_{12})]$	$+ \cos(q_{11})\cos(q_{12})]$
$y_{13} = \sin(q_{11})$	$y_{23} = \sin(q_{12})$

where $a_{11} = \frac{(m_1+m_2)l_1^2}{k_1}$, $a_{12} = \frac{m_2l_1l_2}{k_1}$, $a_{13} = -(m_1 + m_2)l_1g$, $a_{21} = \frac{m_2l_2^2}{k_2}$, $a_{22} = \frac{m_2l_1l_2}{k_2}$, $a_{23} = -m_2l_2g$. Rewrite the equations (10) with these terms:

$$M_1\dot{e}_2 = -C_1e_2 - x_{1d} + e_3 - e_1 + u_2 - y_r\alpha_r \quad (11)$$

The law of control is now defined as:

$$u_2 = -k_2e_2 + x_{1d} + y_r\hat{\alpha}_r \quad (12)$$

The third loop controller error is defined as follows:

$$e_3 = x_3 - u_2 \quad (13)$$

The third virtual control input control rule is proposed as follows:

$$u_3 = -k_3e_3 + \dot{u}_2 \quad (14)$$

At this point, the fourth step error is defined as the relation (15):

$$e_4 = X_4 - u_3 \quad (15)$$

Now derive from the above relationship:

$$\dot{e}_4 = \dot{X}_4 - \dot{U}_3 \quad (16)$$

It can also be written

$$J\dot{e}_4 = J\dot{x}_4 - J\dot{u}_3 \quad (17)$$

By placing the model equations in the above relation, we will have:

$$J\dot{e}_4 = -Bx_4 - k \quad x_3 - x_1 + \tau - J\dot{u}_3 \quad (18)$$

In the above relation, the equation x_3 is inserted:

$$J\dot{e}_4 = -Be_4 + u_3 - ke_3 + u_2 - e_1 - \chi_{1d} + \tau - J\dot{u}_3 \quad (19)$$

By simplifying the above relationship:

$$J\dot{e}_4 = -Be_4 - Bu_3 - ke_3 - ku_2 + ke_1 + kx_{1d} + \tau - J\dot{u}_3 \quad (20)$$

In the above relation, the equation \dot{u}_3 is given by:

$$J\dot{e}_4 = -Be_4 - ke_3 + ke_1 + \tau - J\dot{u}_3 - Bu_3 - k \quad u_2 - X_{1d} \quad (21)$$

In this section, considering a regressor vector for the above parameters, α_j is defined as the parameter vector of unknown and y_j as regressor:

$$-J\dot{u}_3 - Bu_3 - k \quad u_2 - x_{1d} = -y_j\alpha_j \quad (22)$$

The following table is calculated as an adaptive parameter:

Table 2. Flexible robot measurable parameters

Adaptive parameter	Adaptive parameter
y_{11}	y_{21}
$= \dot{q}_{12} - 2\dot{q}_{11} + 2\dot{x}_{1d}$	$= \dot{q}_{22} - 2\dot{q}_{12} + 2\dot{x}_{2d}$
y_{12}	y_{22}
$= -q_{12} + 2k_2q_{11}$	$= -q_{22} + 2k_2q_{12}$
$- 2x_{1d}$	$- 2x_{2d}$
$y_{13} = 2k_2q_{11}$	$y_{23} = 2k_2q_{12}$

Where $a_{11} = J_1$, $a_{12} = B_1$, $a_{13} = K_1$, $a_{21} = J_2$, $a_{22} = B_2$, $a_{23} = K_2$. The law of control is considered as the fourth step:

$$\tau = -k_4e_4 + y_j\hat{\alpha}_j \quad (23)$$

Now the Lyapunov function is defined as the last step:

$$V = V_1 + V_2 + V_3 + V_4 \quad (24)$$

$$V_4 = \frac{1}{2}e_4^T J e_4 + \frac{1}{2}\hat{\alpha}_j^T \Gamma_j^{-1} \hat{\alpha}_j$$

On the other hand, to prove the sustainability of the system, we consider the following matching rules:

$$\dot{\hat{\alpha}}_r = -\Gamma_r e_2 y_r^T \quad (25)$$

$$\dot{\hat{\alpha}}_j = -\Gamma_j e_4 y_j^T$$

Also, according to the Rayleigh-Ritz property, the following relationship exists:

$$-e_1^T k_1 e_1 \leq -\lambda_{\min} \quad k_1 \quad \|e_1\|^2 \quad (26)$$

$$-e_2^T k_2 e_2 \leq -\lambda_{\min} \quad k_2 \quad \|e_2\|^2 \quad (27)$$

$$-e_3^T k_3 e_3 \leq -\lambda_{\min} \quad k_3 \quad \|e_3\|^2 \quad (28)$$

$$-e_4^T k_4 e_4 \leq -\lambda_{\min} \quad k_4 \quad \|e_4\|^2 \quad (29)$$

$$-e_4^T B e_4 \leq -\lambda_{\min} \quad B \quad \|e_4\|^2 \quad (30)$$

According to Young's unequal relationship, the following relationship exists:

$$e_1^T e_2 \leq \|e_1\| \|e_2\| \leq \frac{1}{2} \|e_1\|^2 + \frac{1}{2} \|e_2\|^2 \quad (31)$$

$$e_2^T e_3 \leq \|e_2\| \|e_3\| \leq \frac{1}{2} \|e_2\|^2 + \frac{1}{2} \|e_3\|^2 \quad (32)$$

$$-e_2^T e_1 \leq \|e_2\| \|e_1\| \leq \frac{1}{2} \|e_1\|^2 + \frac{1}{2} \|e_2\|^2 \quad (33)$$

$$e_3^T e_4 \leq \|e_3\| \|e_4\| \leq \frac{1}{2} \|e_3\|^2 + \frac{1}{2} \|e_4\|^2 \quad (34)$$

$$-e_4^T K e_3 \leq \frac{1}{2} \lambda_{\max} K \|e_4\|^2 + \frac{1}{2} \lambda_{\max} K (41) \|e_3\|^2 \quad (35)$$

$$e_4^T K e_1 \leq \frac{1}{2} \lambda_{\max} K \|e_4\|^2 + \frac{1}{2} \lambda_{\max} K \|e_1\|^2 \quad (36)$$

Given the above relationships, the derivative relation of the Lyapunov function is rewritten as follows:

$$\begin{aligned} \dot{V} \leq & -\lambda_{\min}(k_1) \|e_1\|^2 + \frac{1}{2} \|e_1\|^2 + \frac{1}{2} \|e_2\|^2 \\ & -\lambda_{\min}(k_2) \|e_2\|^2 + \frac{1}{2} \|e_2\|^2 + \frac{1}{2} \|e_3\|^2 + \frac{1}{2} \|e_1\|^2 + \\ & \frac{1}{2} \|e_2\|^2 - \lambda_{\min}(k_3) \|e_3\|^2 + \frac{1}{2} \|e_3\|^2 + \frac{1}{2} \|e_4\|^2 \\ & -\lambda_{\min}(B) \|e_4\|^2 + \frac{1}{2} \lambda_{\max}(K) \|e_3\|^2 + \\ & \frac{1}{2} \lambda_{\max}(K) \|e_4\|^2 + \frac{1}{2} \lambda_{\max}(K) \|e_1\|^2 \\ & -\lambda_{\min}(k_4) \|e_4\|^2 \leq 0 \end{aligned} \quad (37)$$

Relationship (37) is rewritten by taking the above coefficients into Relationship Form (38):

$$\dot{V} \leq -\alpha_1 \|e_1\|^2 - \alpha_2 \|e_2\|^2 - \alpha_3 \|e_3\|^2 - \alpha_4 \|e_4\|^2 \leq 0 \quad (38)$$

By considering $\alpha_i > 0$ and based on the Barbalet Lemma and the negation of the derivative of the Lyapunov function, it can be concluded:

$$\lim_{t \rightarrow \infty} e(t) = 0$$

Therefore, system stability is guaranteed and in this case, the system error also converges to zero.

4. Simulation

This paper aims to track the sinusoidal path at both angles of the robot joint. In this paper, a robust controller design is simulated in MATLAB software with sample time $T_s=0.01s$. For evaluating proposed method, controller designed for a non-holonomic mobile robot in [11] is considered and results of controller for our model are shown in Fig. 2 and 3. The proposed backstepping controller coefficients are set to $[k_1 \ k_2 \ k_3 \ k_4] = [4 \ 4 \ 30 \ 1]$ based on trial and error. In this simulation, the model parameters are given in Table 3.

Table 3. Parameters of the FJR Two-Joint Robot Model

Unit	Value	Parameter
Nm/rad	100	K_1
Nm/rad	100	K_2
Nms/rad	0.9	B_2
Nms/rad	0.9	B_2
Kg	0.5	m_1
Kg	0.5	m_2
M	1	L_1
M	1	L_2

In this simulation, the optimal path for the first and second joint angles are $\sin(t)$ and $1-\cos(t)$, respectively. The initial values of the angles of the robot are set to zero. The simulation results with the conditions expressed in the form of two outputs q_1 and q_2 are flexible joints and the control inputs in the form of τ_1 and τ_2 are shown in Figs 2 and 3.

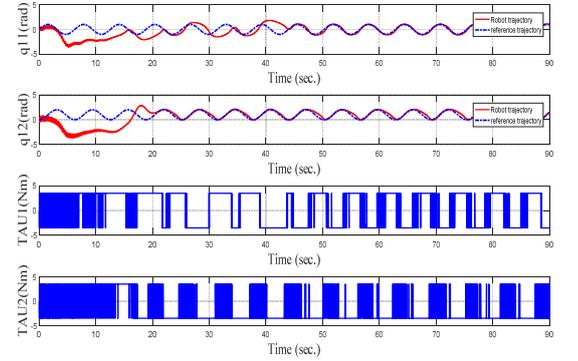


Fig. 2. q_{11} and q_{12} using nonlinear controller [11] and controller inputs

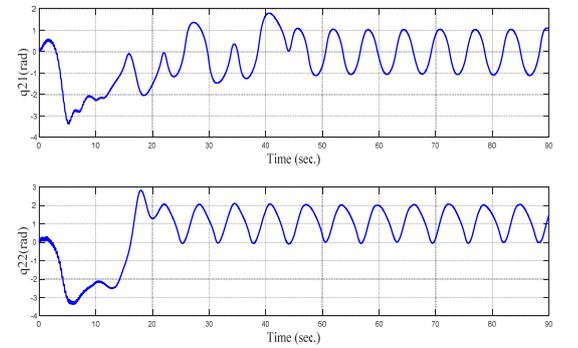


Fig. 3. q_{21} and q_{22} using the nonlinear controller of [11]

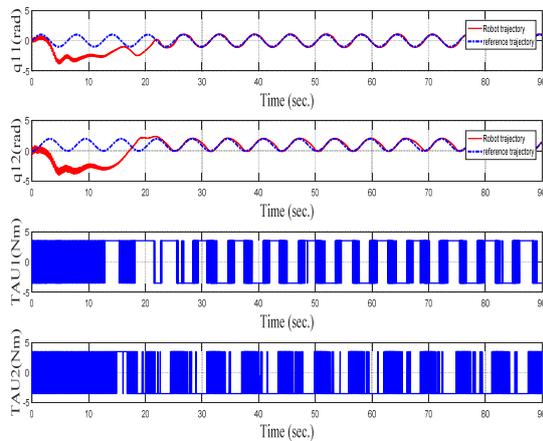


Fig. 4. q_{11} and q_{12} using backstepping controller and controller inputs

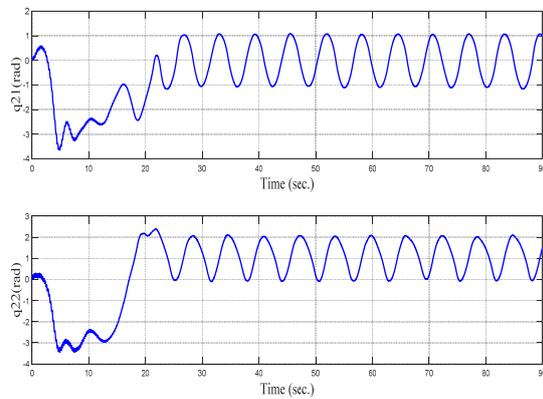


Fig. 5. q_{21} and q_{22} using the backscatter controller

According to the method results, the first and second joint angular followed the desired sinusoidal path by considering the adaption parameter in 25s and after that, without any overshoot and error, dynamics of the model have been stabled, while in the method of [11] there are fluctuations on the output waveform which is not desirable. Based on these results, it is quite clear that without the main value of the model parameter, the controller acts properly.

5. Conclusion

In this paper, a two-degree-of-freedom flexible robot system is modeled as a nonlinear system and the control law is designed based on the adaptive backstepping control method. In the nonlinear model, finding the fixed value for these parameters is not easy and requires trial and error or the use of parameter estimation methods. It has been theoretically demonstrated that the proposed controller can optimize the arm in the presence of unknowing parameters by designing control inputs through consecutive feedbacks despite minor errors. By using the backstepping control

method, the control input is designed. The proposed control law is simulated and analyzed in a flexible two-joint robot in MATLAB software. The results of the simulations show the ability of the proposed control law to follow the desired path for the angles of the robot joints and to bring the tracking error to zero, and according to Lyapunov's theory, the stability of the model is guaranteed.

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